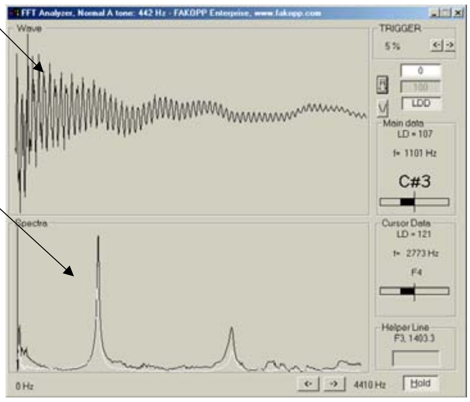
Vibration Methods

Basic idea

- Best conditions for vibration test are:
  - support sample at the nodal points
  - hit sample at amplitude maximum,
  - hitting direction is important
  - microphone locate at max amplitude location

Fourier Transformation

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-i2\pi ft} dt \qquad i = \sqrt{-1}$$



Discrete Fourier Transformation

$$h(kT) \qquad k = 0,1,2,3,......., N-1$$

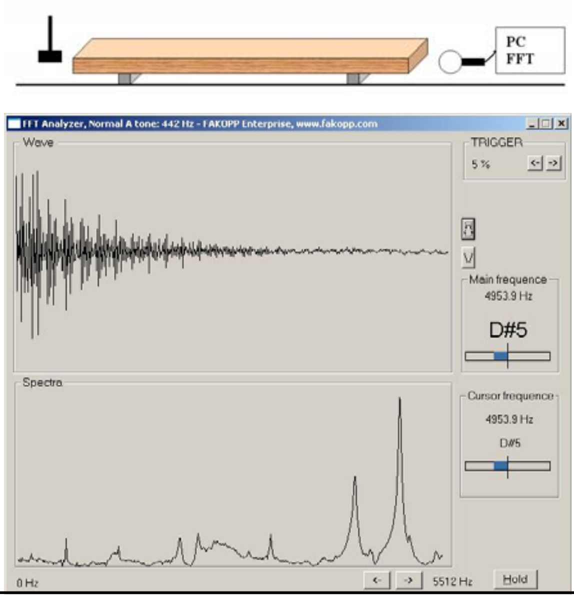
T: time between two samples, $T=1/f_{sampling}$

$$H\left(\frac{n}{NT}\right) = \sum_{k=0}^{N-1} h(kT) e^{-i2\pi n\frac{k}{N}} \qquad n = 0,1,2,3,.....N-1$$

Fast Fourier Transformation:

h function is identical, H is almost the same, but computation time is shorter.

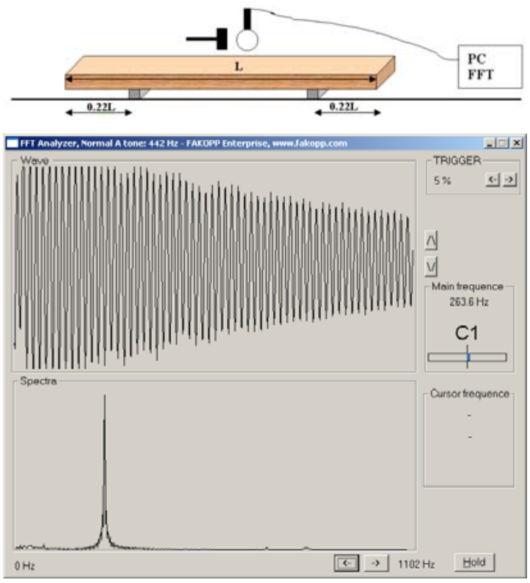
Dynamic MOE determination by longitudinal vibration



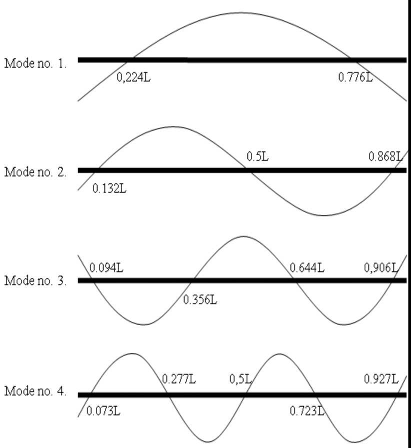
$$MOE_{dyn,long} = \rho V^2$$

$$V = 2Lf$$

Dynamic MOE determination by bending vibration



$$MOE_{dyn,bending} = \left(\frac{f_n}{C_n}\right)^2 \frac{mL^3}{I}$$



Timoshenko beam theory

- The effect of shear is included.

$$EI\frac{\partial^4 r}{\partial x^4}+\rho A\frac{\partial^2 r}{\partial t^2}-\rho I\left(1+\frac{E}{\beta G}\right)\frac{\partial^4 r}{\partial x^2\partial t^2}+\frac{\rho^2 I}{\beta G}\frac{\partial^4 r}{\partial t^4}=0$$

where: *b* - shear factor (1/1.2 for prismatic beams),
r - displacement,
x - longitudinal coordinate,
t - time,
A - cross section,
ρ - density,
I - moment of inertia,
E - bending modulus of elasticity, MOE
G - shear modulus.

Solution is available by iteration technique only.

Solution of the Timoshenko equation

- Evaluation software presented by Dr. Chui, Canada. Input data are:
  - length
  - thickness
  - density
  - bending vibration frequency, mode 1
  - bending vibration frequency, mode 2

Demonstration……… E.exe

Euler beam theory

Neglecting the shear Timoshenko theory turns to Euler beam theory

$$E = \left(\frac{f_n}{C_n} \right)^2 \frac{mL^3}{I}$$

where: f_n - frequency measured in the *n*th mode,

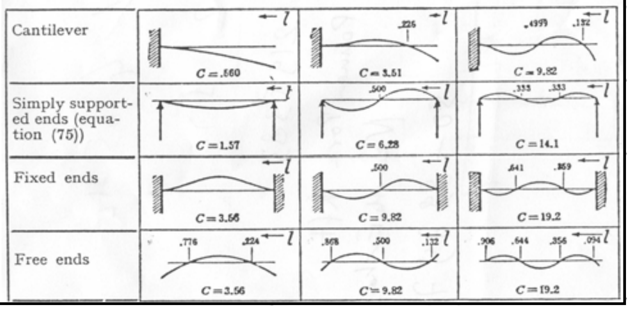
C_n - mode factor (see Figure),

m - specimen mass.

E - MOE

I - Inertia

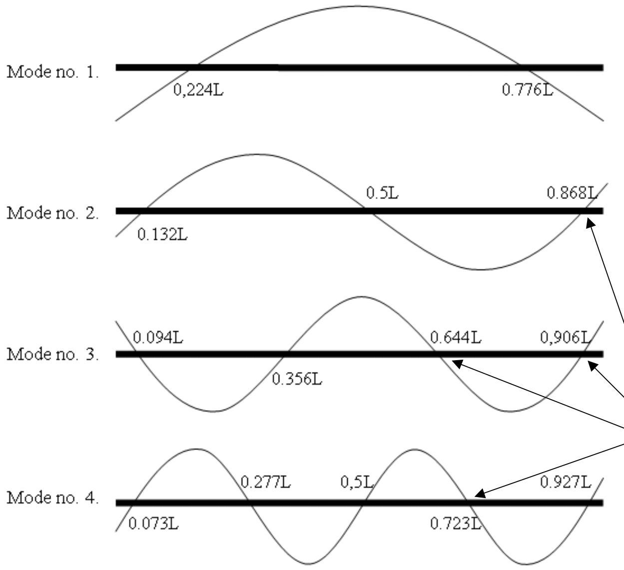
Mode 1. *2.* *3.*



Influencing factors are:

- Geometry: Nodal distance/thickness
- Damping
- Static-dynamic correction

Nodal distance



Nodal distance is the distance between two neighbouring nodal points.

Nodal distance / beam height

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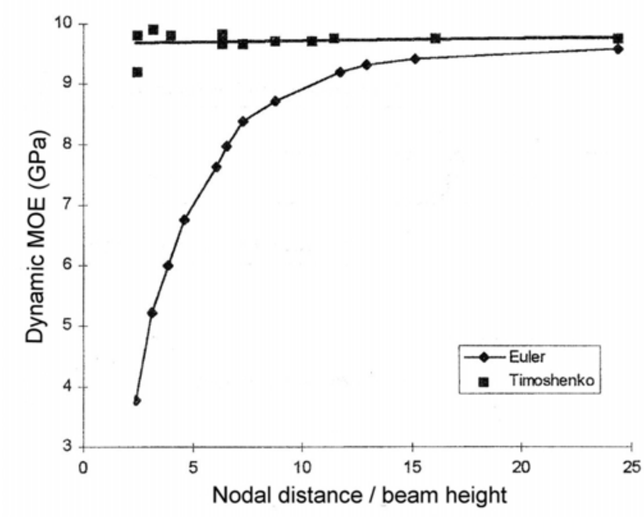


Effect of damping

- The measured frequency (f) and the not damped vibration frequency (f_o) is different

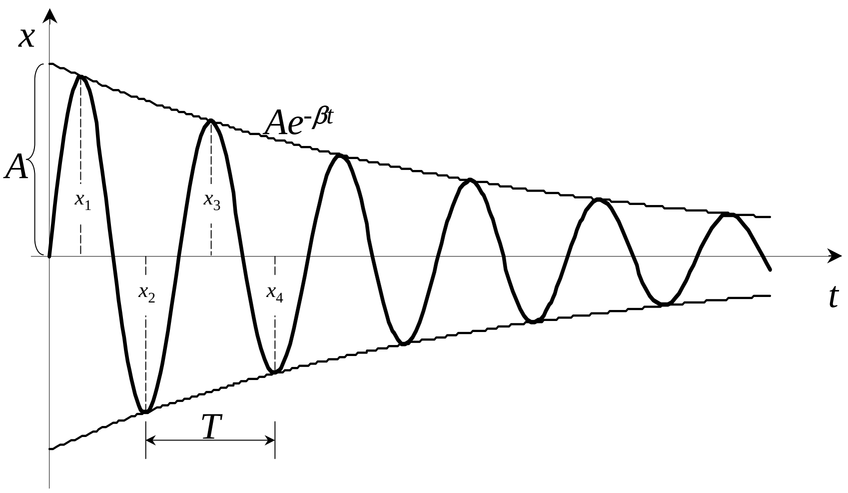
$$f_0 = f\sqrt{1+\frac{\Lambda^2}{4\pi^2}}$$

Λ is the logarithmic decrement: $\Lambda = \beta/f$

β is damping coefficient

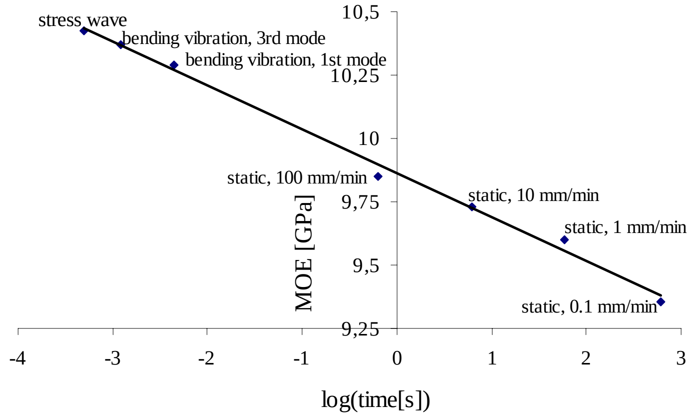
Logarithmic decremet for wood is 0,1-0,01, the damping correction is minor, less than 0,2%

Damping coefficient: β

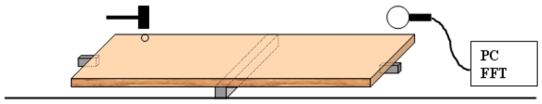


Static - dynamic correction

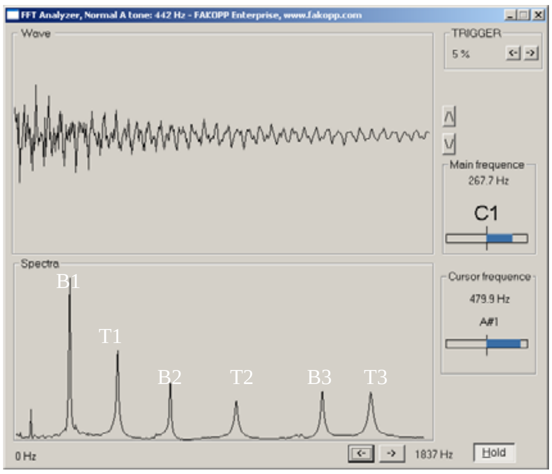
- One order of magnitude difference in the characteristics time causes a 1.7% change in the measured MOE value.



Determination of shear modulus by torsion vibration



$$G_{dyn,torsion} = \left(\frac{2Lf_n}{n}\right)^2 \frac{\rho I_p}{K_t}$$

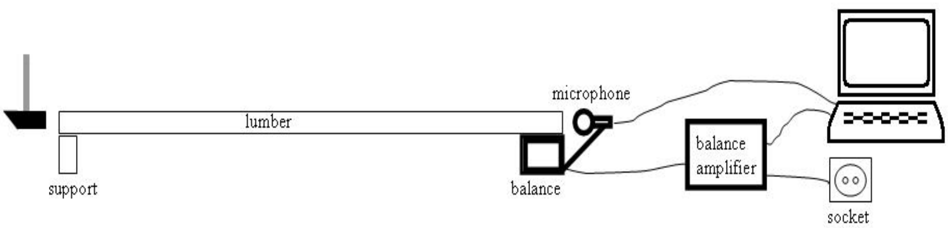


Evaluation chart

	A	B	C	D	E	F	G	H
1	**Dimensions**				**Determination of**			
2	width	45,3	mm	Modulus of elasticity (MOE)			Shear modulus (G)	
3	thickness	24,1	mm	Mass	0,6	kg		
4	length	0,67	m	I	5,28E-08	m^4		
5								
6	LONGITUDINAL VIBRATION			BENDING VIBRATION			TORSIONAL VIBRATION	
7	Frequency							
8	f	3748	Hz	Frequency	Mode no.	MOE_{bend}	Frequency	G
9				273,2	1	**20,1**	823	**1,70**
10	ρ	820	kg/m^3	734,2	2	**19,1**	Hz	**GPa**
11	Velocity	5022	m/s	1386	3	**17,7**		
12				2195	4	**16,3**		
13	**MOE_{long}**	**20,7**	**GPa**	Hz		**GPa**	**MOE/G**	**11,9**

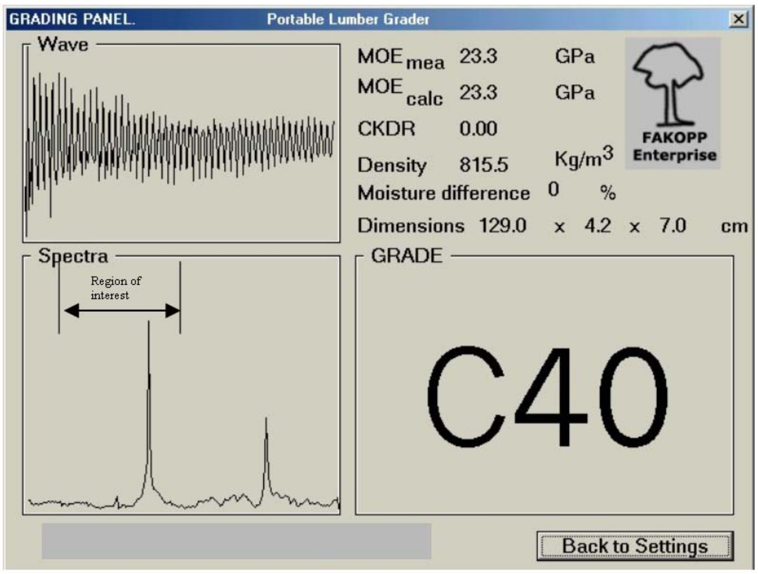
MOE and G evaluation example.xls

Portable Lumber Grader setup



Longitudinal vibration, Portable Lumber Grader (PLG)

PLG screen



PLG algorithm

$$MOE_{mea} = \rho(2lf)^2 \, 0.92(1 + u/50)$$

ρ: density

l: length

f: longitudinal vibration frequency

u: moisture different = actual – service condition

$$MOE = MOE_{mea} - 6.2CKDR$$

Concentrated Knot Diameter Ratio

The knot diameter is a distance between the two tangential lines parallel to arises (longitudinal direction) of a lumber surface in which the knot exists. If a knot diameter not less than 2.5 times as much as its smallest diameter, it shall be considered to have one half of its actual measured diameter. The knot diameter ratio (KDR) is a percentage of the diameter of a knot to the width of a lumber surface in which it exists. The concentrated KDR (CKDR) is the sum of KDR concerning the knots existing in any 15 cm length of a piece of the lumber. The highest - considering 4 faces - CKDR represents the piece of lumber. The CKDR value is between 0 and 1.

PLG decision table

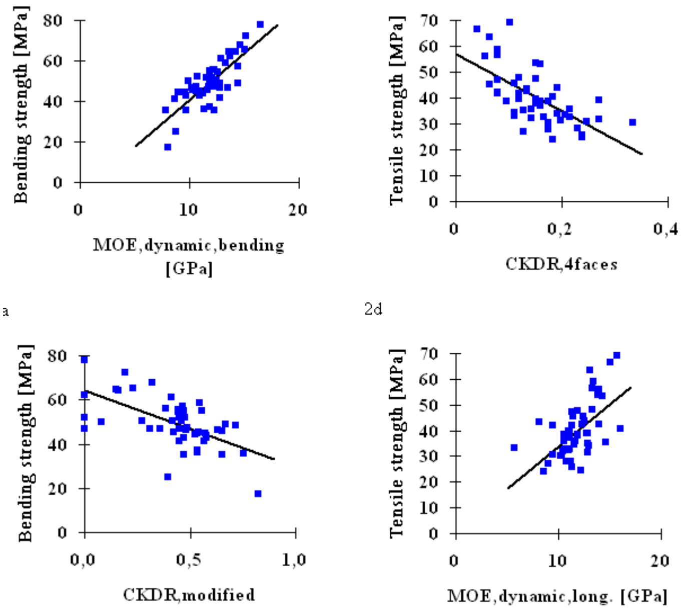
For coniferous species group the following table is used:

Grade	C14	C16	C18	C22	C24	C27	C30	C35	C40
MOE (GPa)	7	8	9	10	11	12	12	13	14
ρ (kg/m³)	290	310	320	340	350	370	380	400	420

For deciduous species group the following table is used:

Grade	D30	D35	D40	D50	D60	D70
MOE (GPa)	10	10	11	14	17	20
ρ (kg/m³)	530	560	590	650	700	900

Example, 2 by 4 material



Demonstration: PLG and rapture test