Sólido deformable: cables

Mariano Vázquez Espí

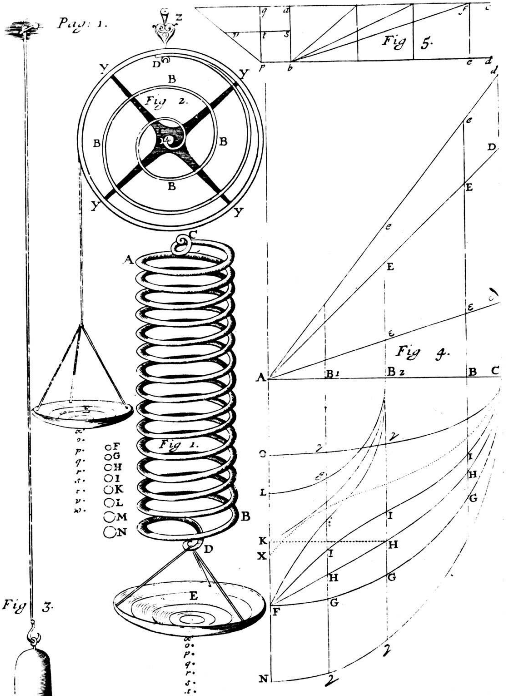
Madrid (España), 11 de febrero de 2010.

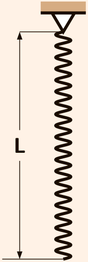
Robert Hooke (1635–1703)
—Físico, astrónomo y naturalista

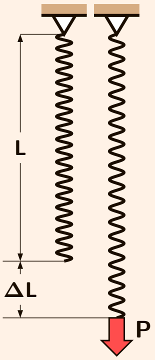
> **Entre otras cosas, introdujo el concepto de *célula* y analizó la anatomía de los insectos.**

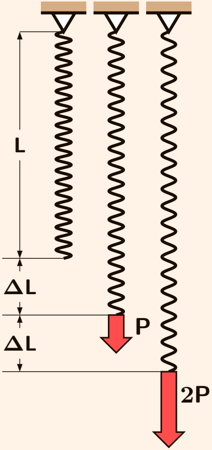
Thomas Young (1773–1829)
—Físico y médico

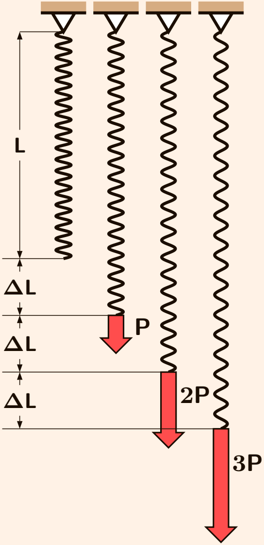
> **Entre otras cosas, introdujo el concepto moderno de *energía* y contribuyó a descifrar la escritura jeroglífica egipcia.**

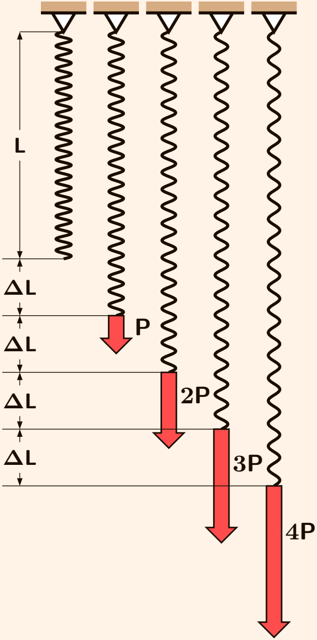


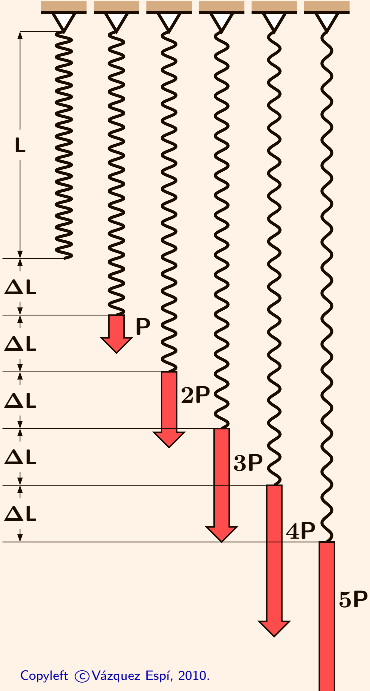

L

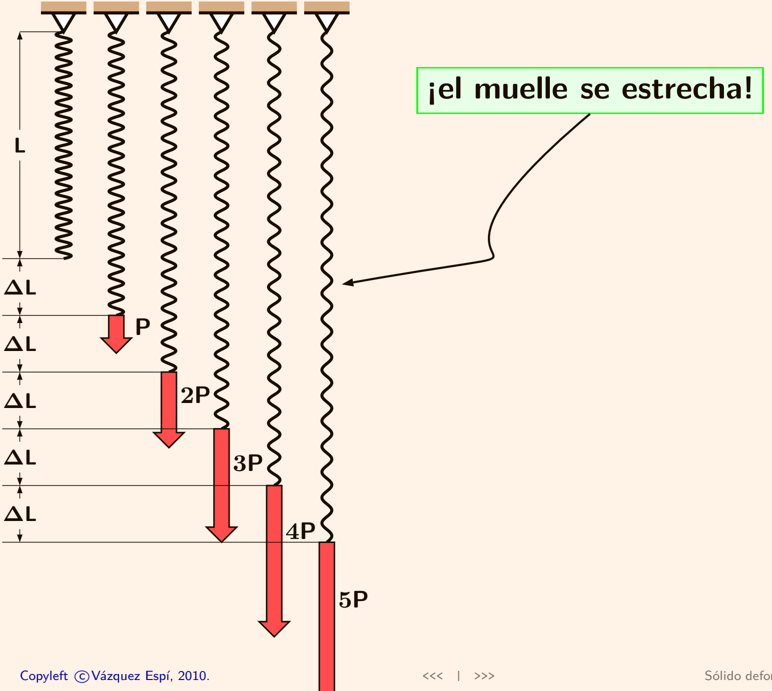

L
ΔL
P

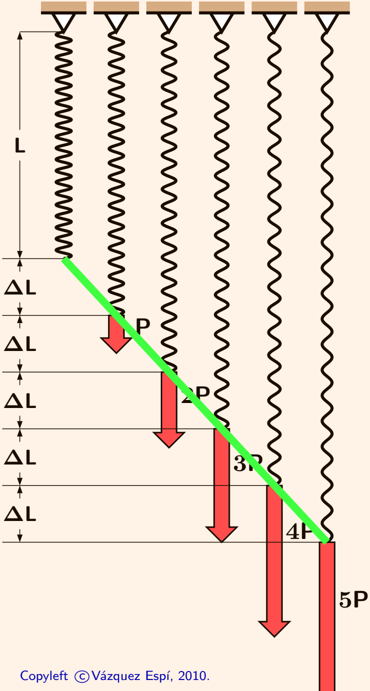

L
ΔL
ΔL
P
2P

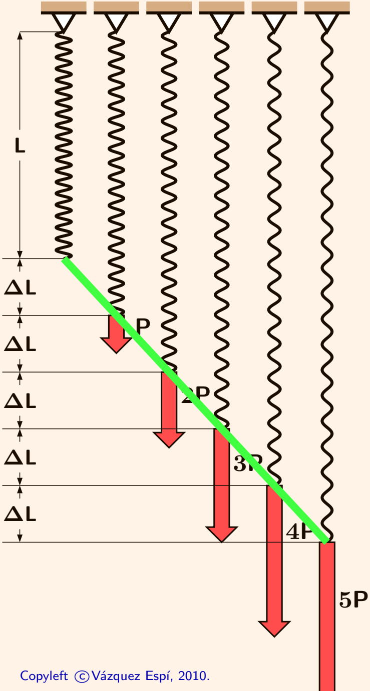

L
ΔL
ΔL
ΔL
P
2P
3P


L
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P

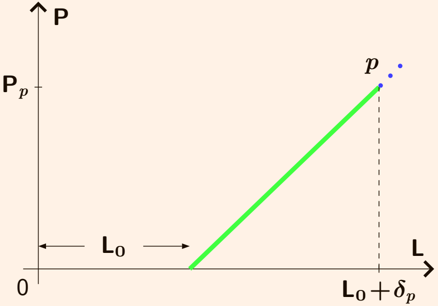

L
ΔL
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P
5P


¡el muelle se estrecha!
L
ΔL
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P
5P


L
ΔL
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P
5P

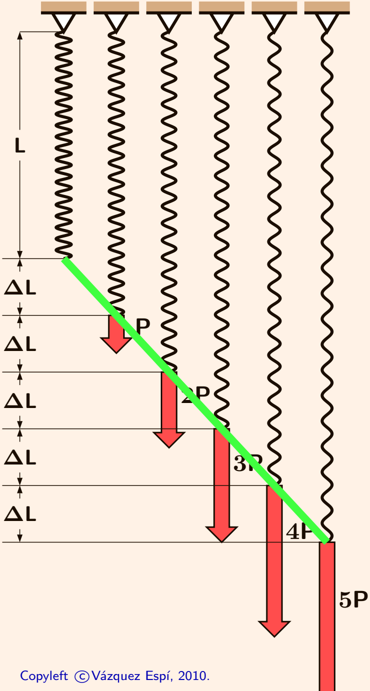


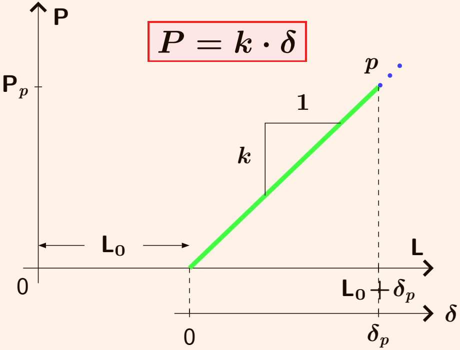
L
ΔL
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P
5P

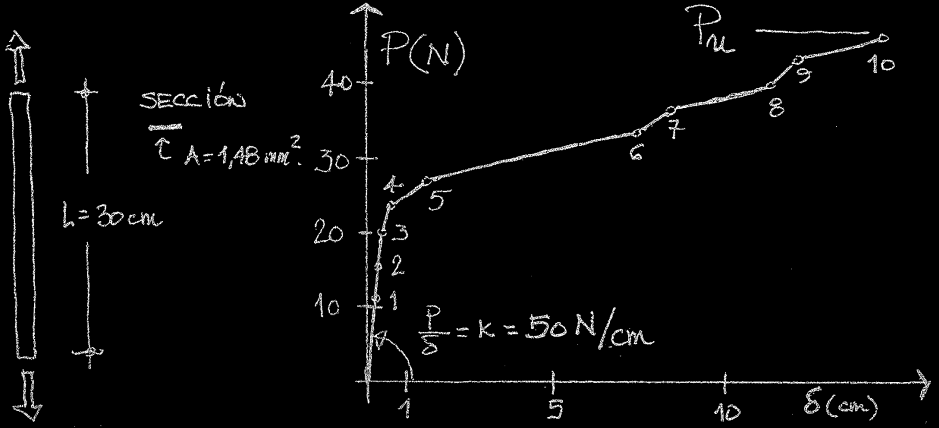


P
P_p
p
L_0
L
0
$L_0 + \delta_p$

$$P = k \cdot \delta$$

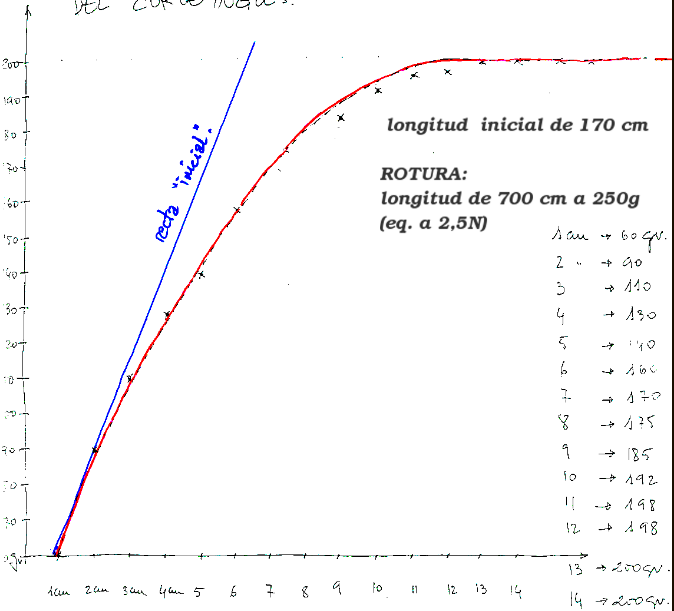






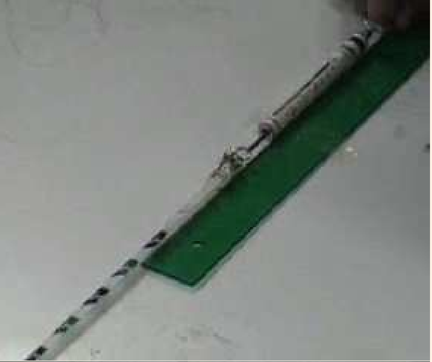
P(N)
Pu
SECCIÓN
A = 1,48 mm².
L = 30 cm
40
30
20
10
1
2
3
4
5
6
7
8
9
10
P/δ = k = 50 N/cm
1
5
10
δ (cm)

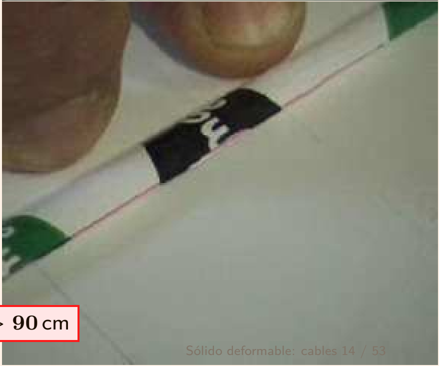
ENSAYO DE TRACCIÓN CON PAPEL DEL CORTE INGLÉS.



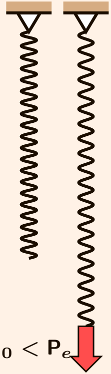
longitud inicial de 170 cm

ROTURA:
longitud de 700 cm a 250g
(eq. a 2,5N)

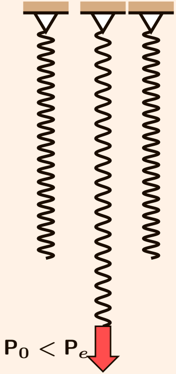


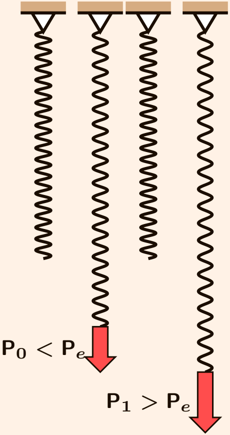


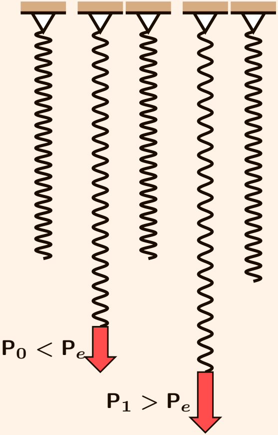
$$\mathbf{L_0 = 90\,\text{cm},\ \delta_u > 90\,\text{cm}}$$

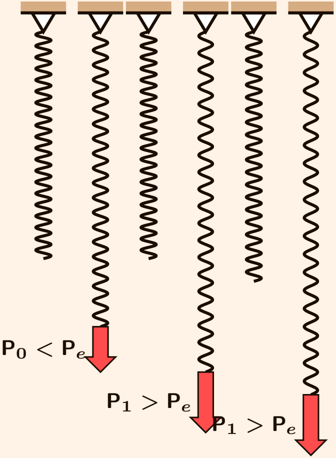


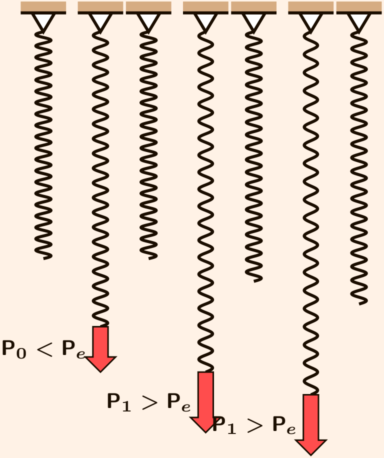
$\mathbf{P}_0 < \mathbf{P}_e$

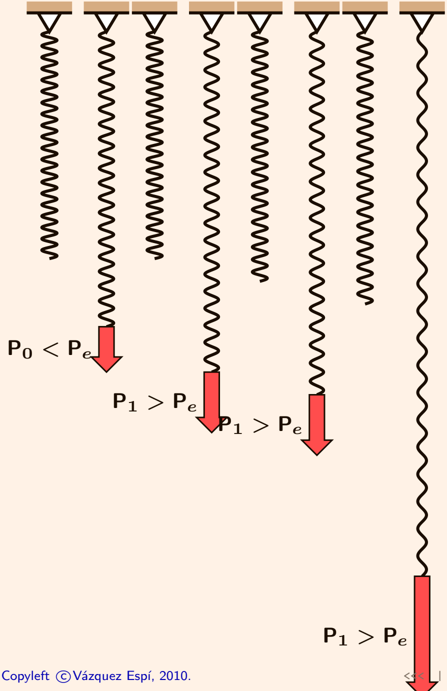

$\mathbf{P}_0 < \mathbf{P}_e$

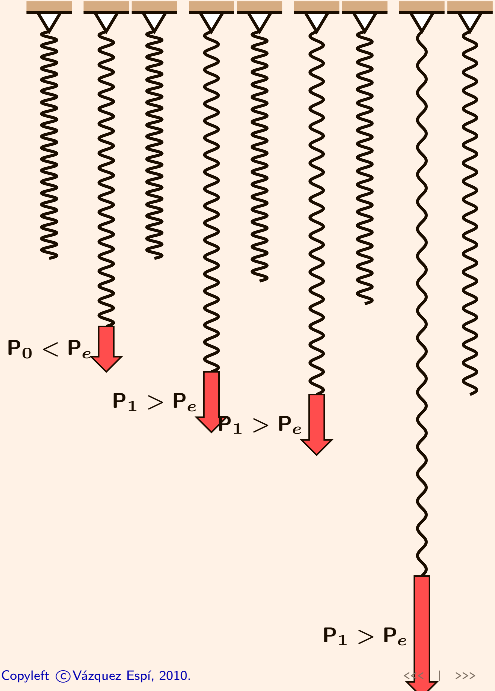

$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$

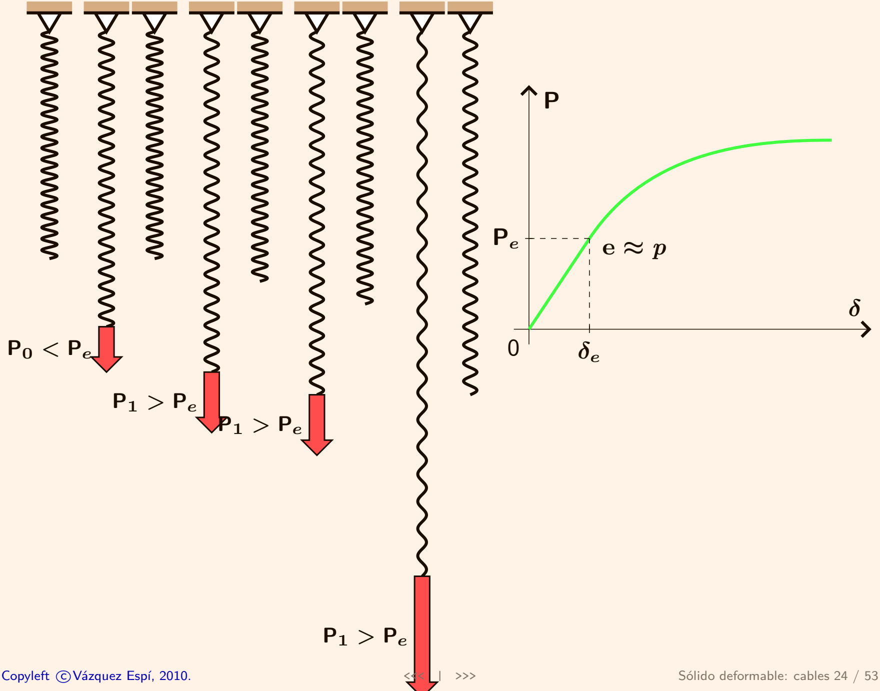

$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$


$P_0 < P_e$
$P_1 > P_e$
$P_1 > P_e$


$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$

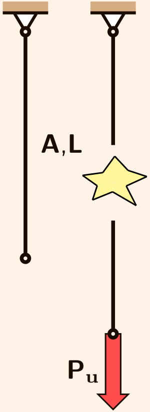

$P_0 < P_e$
$P_1 > P_e$
$P_1 > P_e$
$P_1 > P_e$

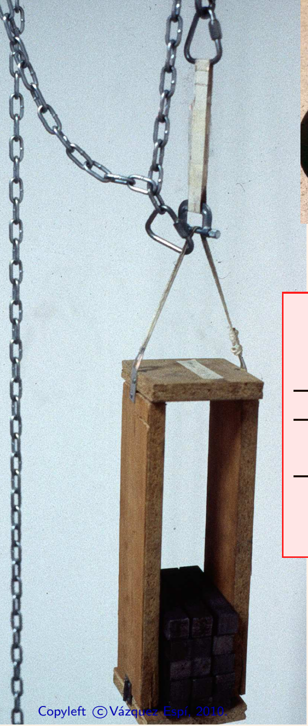

$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$

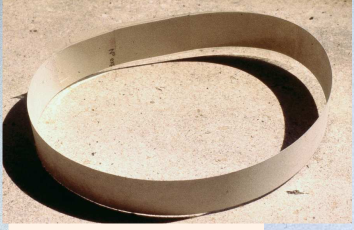


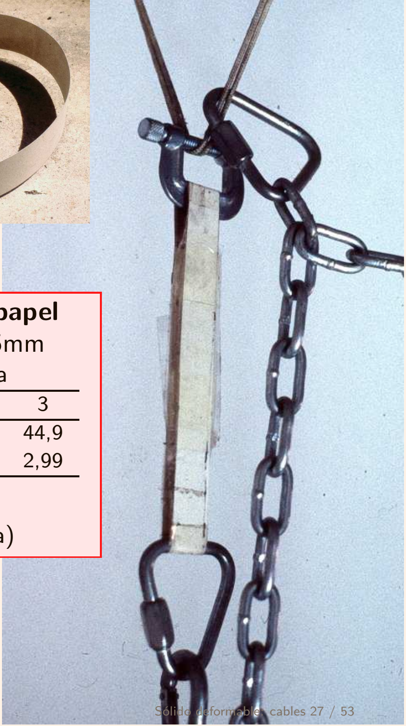
$P_0 < P_e$
$P_1 > P_e$
$P_1 > P_e$
$P_1 > P_e$
P
P_e
$e \approx p$
δ
0
δ_e


A, L


A, L
P u







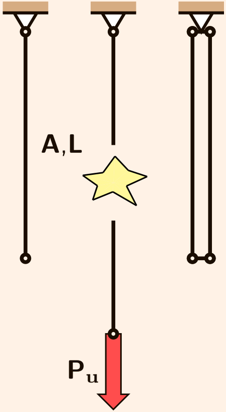
Ensayo de cinta de papel

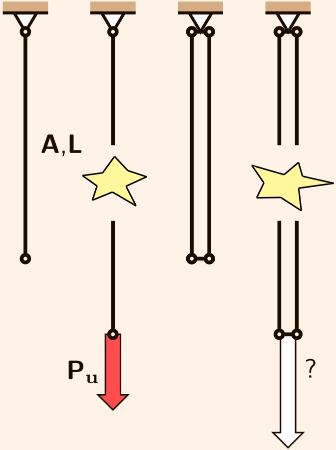
ancho de la cinta: 15mm

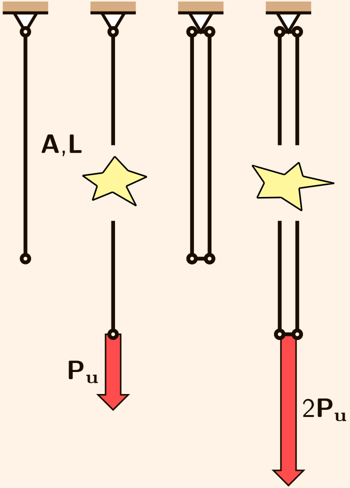
Valores en la rotura

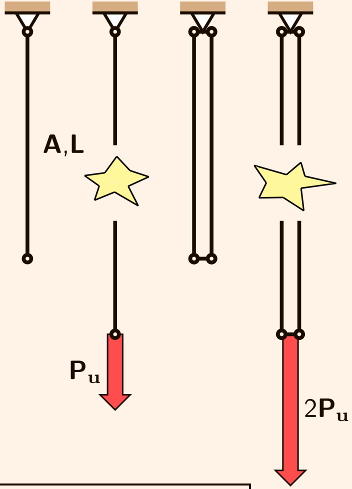
ensayo	1	2	3
P/2 (N)	55,7	56,3	44,9
f (N/mm)	3,71	3,75	2,99

$\mathbf{f_u} \approx \mathbf{2,99}$ N/mm
(100 % de confianza)

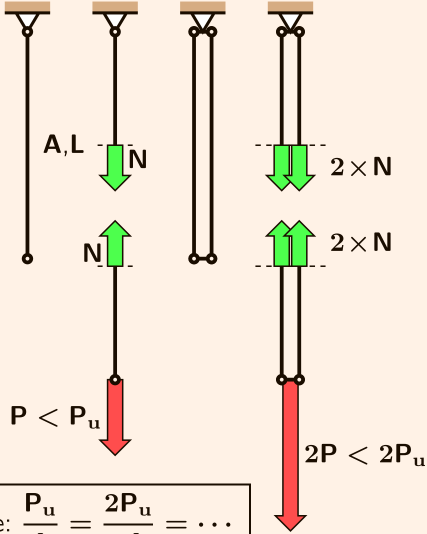

A, L
P_u


A,L
P
u
?


A,L
P_u
$2P_u$



$$\text{cte: } \frac{\mathbf{P_u}}{\mathbf{A}} = \frac{\mathbf{2P_u}}{\mathbf{2A}} = \cdots$$

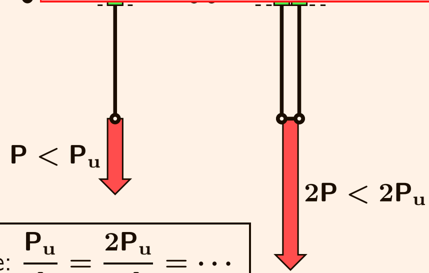


$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$

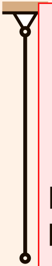
Tensión

$$\bar{\sigma} = \frac{N}{A} \qquad \left(\text{en este caso } \frac{P}{A}\right)$$

Fuerza por unidad de área de la sección de la barra. N/m^2(un 'pascal'), N/mm^2, kN/mm^2, etc.



$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$



Tensión

$$\bar{\sigma} = \frac{N}{A} \qquad \left(\text{en este caso } \frac{P}{A}\right)$$

Fuerza por unidad de área de la sección de la barra. N/m^2(un 'pascal'), N/mm^2, kN/mm^2, etc.

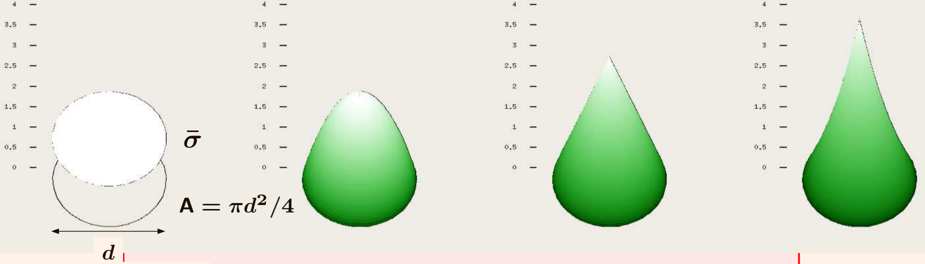
Equilibrio:

$$N = \int \sigma(x, y)\, dA$$

$$\bar{\sigma} = \frac{\int \sigma(x, y)\, dA}{\int dA} = \frac{N}{A}$$

P

cte: $\frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$



$\sigma_{\max} =$
$\sigma_{\min} = \bar{\sigma}$

ibrio:

$\sigma_{\max} = 2\bar{\sigma}$

$\sigma_{\max} = 3\bar{\sigma}$

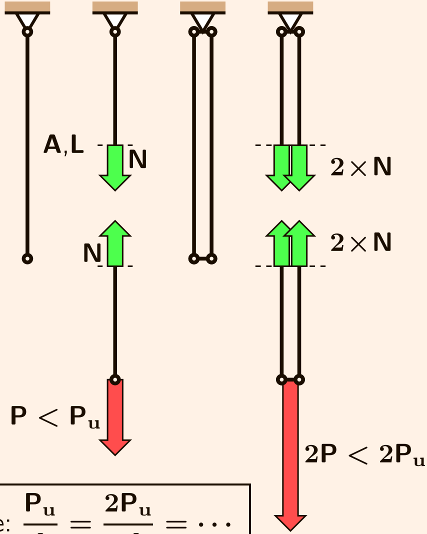
$\sigma_{\max} \approx 3{,}94\bar{\sigma}$

$$\mathsf{N} = \int \sigma(x,y)\,\mathrm{d}\mathsf{A}$$

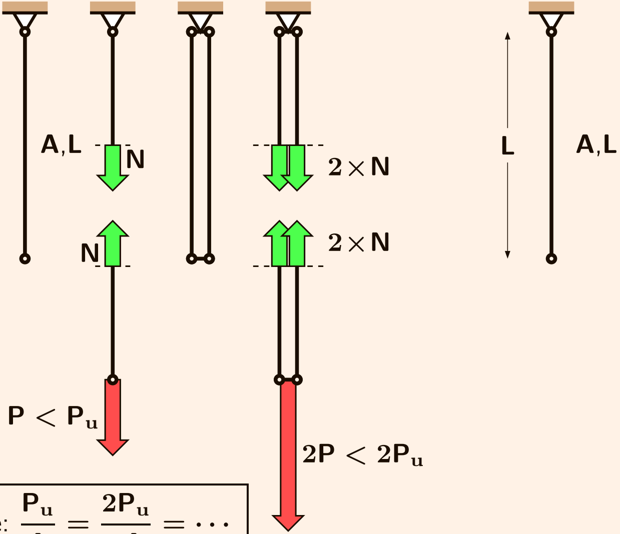
$$\bar{\sigma} = \frac{\int \sigma(x,y)\,\mathrm{d}\mathsf{A}}{\int \mathrm{d}\mathsf{A}} = \frac{\mathsf{N}}{\mathsf{A}}$$

P

cte: $\dfrac{\mathsf{P}_u}{\mathsf{A}} = \dfrac{\;\cdot\;\mathsf{P}_u}{2\mathsf{A}} = \cdots$



$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$



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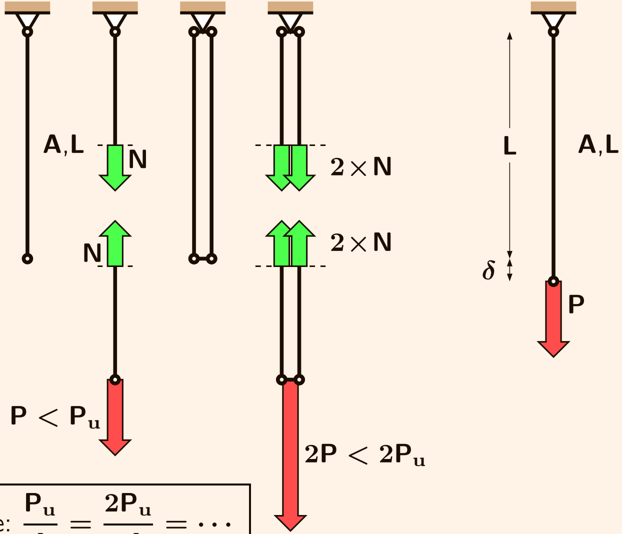

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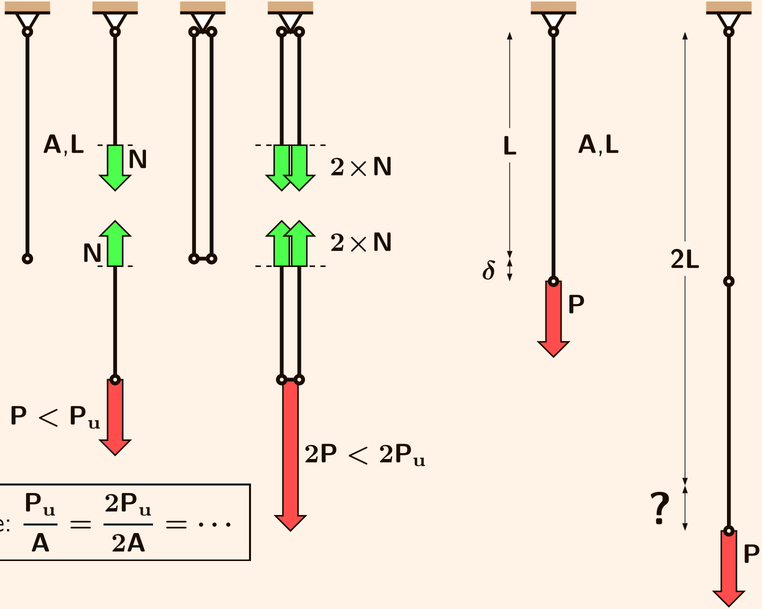

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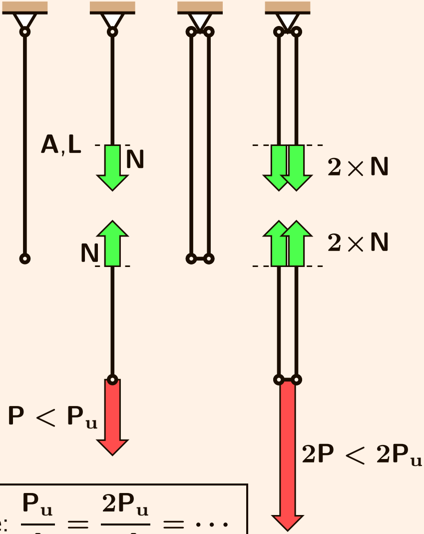
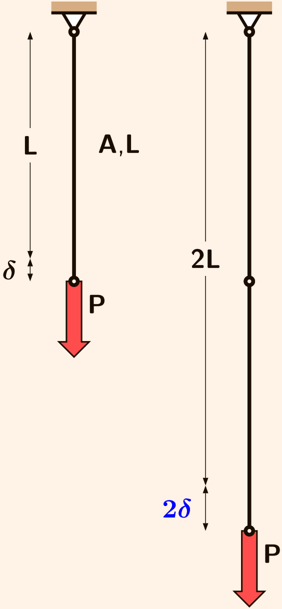
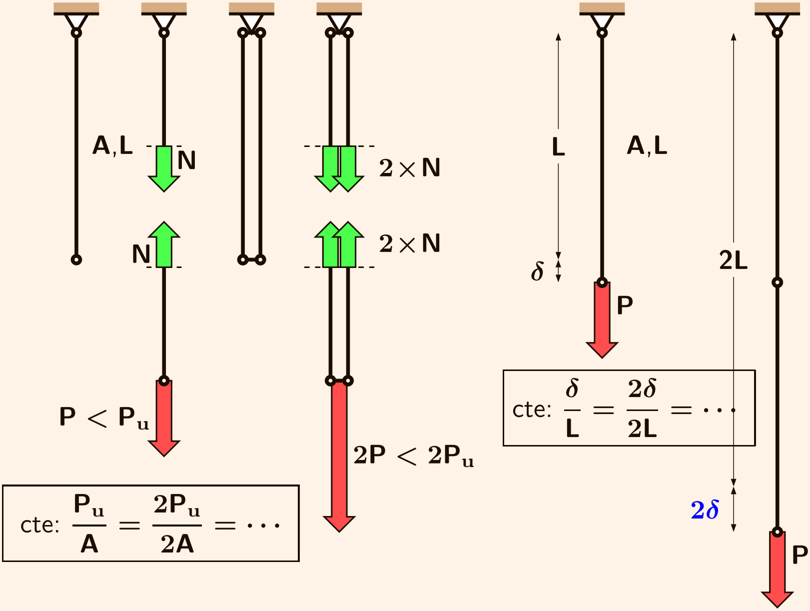


$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$

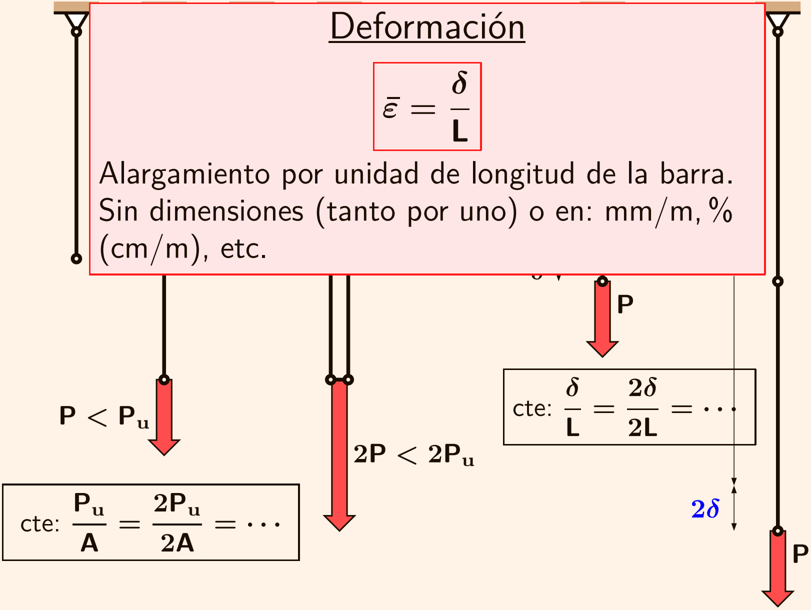


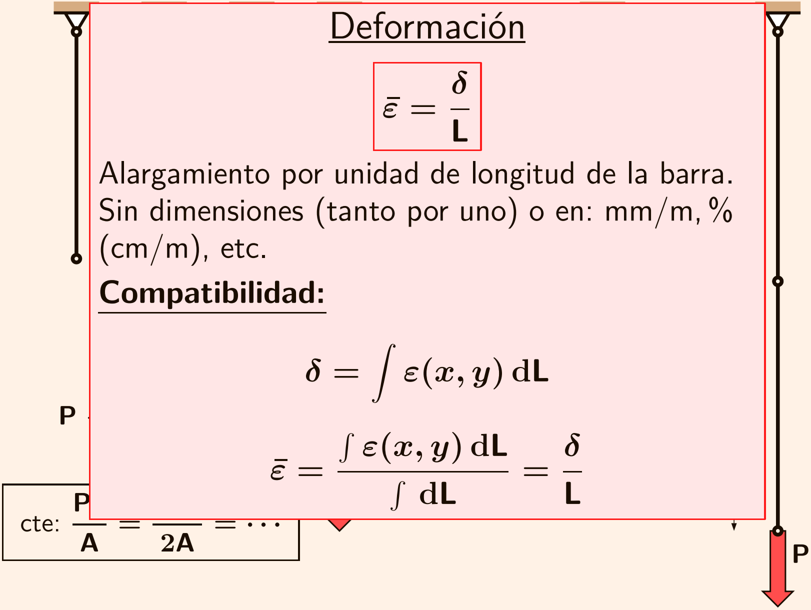

A,L
N
N
2 × N
2 × N
P < Pu
2P < 2Pu
cte: Pu/A = 2Pu/2A = ···
L
δ
A,L
P
cte: δ/L = 2δ/2L = ···
2L
2δ
P

Deformación

$$\bar{\varepsilon} = \frac{\delta}{\mathbf{L}}$$

Alargamiento por unidad de longitud de la barra. Sin dimensiones (tanto por uno) o en: mm/m, % (cm/m), etc.





Deformación

$$\bar{\varepsilon} = \frac{\delta}{L}$$

Alargamiento por unidad de longitud de la barra. Sin dimensiones (tanto por uno) o en: mm/m, % (cm/m), etc.

Compatibilidad:

$$\delta = \int \varepsilon(x, y)\, \mathrm{d}L$$

$$\bar{\varepsilon} = \frac{\int \varepsilon(x, y)\, \mathrm{d}L}{\int\ \mathrm{d}L} = \frac{\delta}{L}$$

$$\delta = \int \varepsilon(x,y)\,\mathrm{d}\mathbf{L}$$

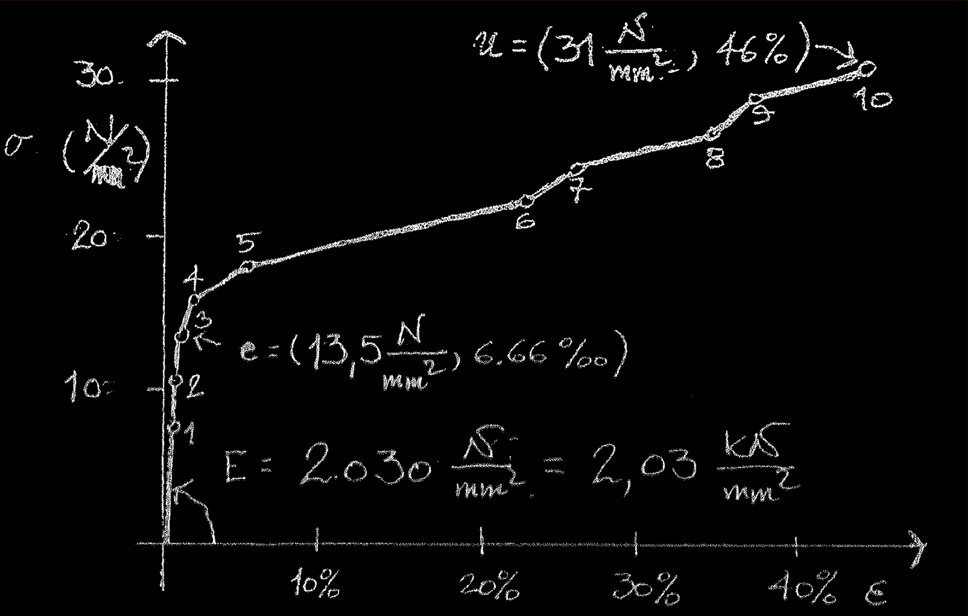
$$\bar{\varepsilon} = \frac{\int \varepsilon(x,y)\,\mathrm{d}\mathbf{L}}{\int \,\mathrm{d}\mathbf{L}} = \frac{\delta}{\mathbf{L}}$$

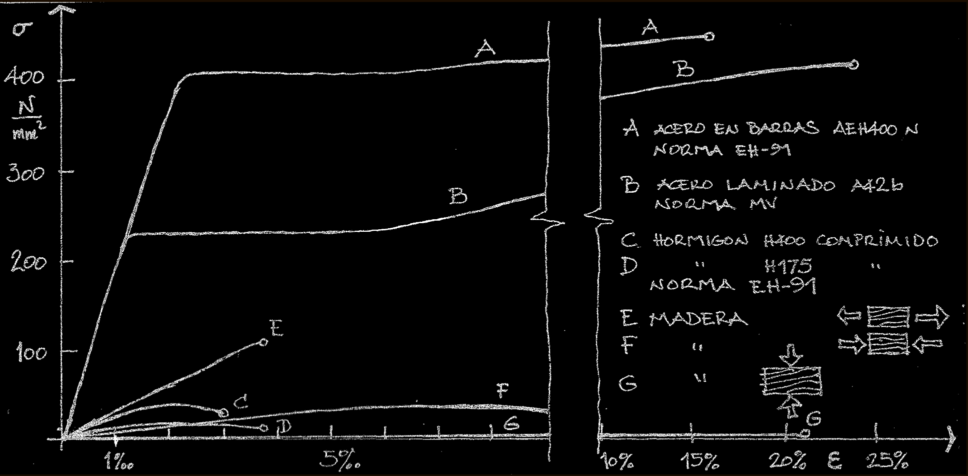
P

cte: $\frac{\mathbf{P}}{\mathbf{A}} = \frac{}{\mathbf{2A}} = \cdots$

P

video: http://www.aq.upm.es/Departamentos/Estructuras/e96-290/doc/


u = (31 N/mm², 46%)
30
σ (N/mm²)
20
10
5
4
3
2
1
6
7
8
9
10
e = (13,5 N/mm², 6.66 ‰)
E = 2.030 N/mm² = 2,03 kN/mm²
10%
20%
30%
40%
ε


σ
400
N/mm²
300
200
100
A
B
E
C
D
F
G
1‰
5‰
10%
15%
20%
ε
25%
A ACERO EN BARRAS AEH400 N
NORMA EH-91
B ACERO LAMINADO A42b
NORMA MV
C HORMIGON H400 COMPRIMIDO
D " H175 "
NORMA EH-91
E MADERA
F "
G "

Marco Polo describe un puente piedra a piedra.

¿Pero cual es la piedra que sostiene el puente? —pregunta Kublai Kan.

El puente no está sostenido por esta o aquella piedra, —responde Marco— sino por la línea del arco que forman.

Kublai Kan queda silencioso, reflexionando. De repente, dice:— ¿Por qué me hablas entonces de las piedras? Es sólo el arco lo que me importa.

Polo responde:— Sin piedras no habría arco.

Italo Calvino

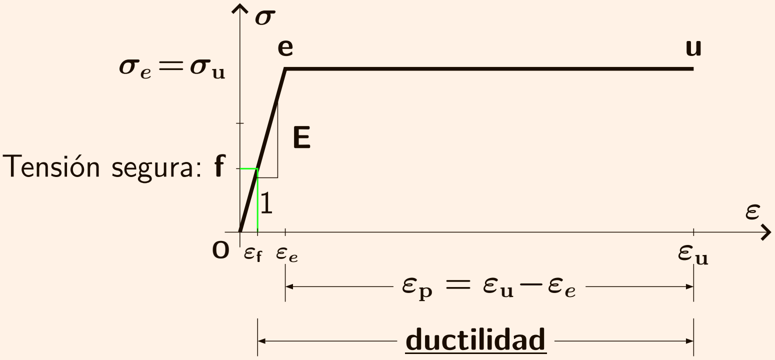
- Comportamiento del material
  - Ley de Hooke: $\mathbf{P} = \mathbf{K}\delta$ si $\mathbf{P} < \mathbf{P}_p$; en otro caso:
  - Plasticidad 'perfecta': $\mathbf{P} = \mathbf{P_u}$ si $\delta < \delta_\mathbf{u}$
- Mientras no se produzca la rotura:
  - Equilibrio: $\mathbf{N} = \mathbf{P}$ (en el caso de cables verticales) y $\sigma = \dfrac{\mathbf{N}}{\mathbf{A}}$ (es decir que $\mathbf{N} = \sigma\mathbf{A}$)
  - Compatibilidad: $\delta = \varepsilon\mathbf{L}$ (y también $\varepsilon = \dfrac{\delta}{\mathbf{L}}$)

En el periodo proporcional:

$$\mathbf{K} = \frac{\mathbf{N}}{\delta} = \frac{\sigma\mathbf{A}}{\varepsilon\ell} = \frac{\mathbf{EA}}{\mathbf{L}} \qquad \sigma = \mathbf{E}\varepsilon$$

Modelo elasto-plástico perfecto de los materiales

$$\boldsymbol{\sigma}(\varepsilon) = \begin{cases} \mathbf{E}\varepsilon & \text{si } \varepsilon \leq \varepsilon_e \\ \boldsymbol{\sigma}_{\mathbf{u}} & \text{si } \varepsilon_e < \varepsilon \leq \varepsilon_{\mathbf{u}} \\ 0 & \text{si } \varepsilon_{\mathbf{u}} < \varepsilon \end{cases}$$



Características importantes de los materiales en el caso de los edificios:

- **Ductilidad:** cuanto mayor deformación antes de la rotura, **¡mejor!**

  (cuanto menor, mayor margen de seguridad γ habrá que adoptar)

- **Fiabilidad**

  (cuanto menor, mayor margen de seguridad γ habrá que adoptar)

- **Coste físico específico**: energía fósil incorporada (*embodied energy*), emisiones contaminantes, etc.

$$\left\{\frac{\mathcal{C}}{\mathbf{E}}, \frac{\mathcal{C}}{\mathbf{f}}, \dots\right\}$$

¡Cuánto menor, mejor!

Modelo ‘cable’

En el estado proporcional, sin superar el límite ‘elástico’:

$$\varepsilon = \frac{\delta}{\mathsf{L}}; \qquad \sigma = \mathsf{E}\varepsilon; \qquad \mathsf{N} = \sigma\mathsf{A}; \qquad \text{si } \varepsilon \leq \varepsilon_e.$$

$$\mathsf{K}_{\text{cable}} = \frac{\mathsf{N}}{\delta} = \frac{\sigma\mathsf{A}}{\varepsilon\mathsf{L}} = \frac{\mathsf{EA}}{\mathsf{L}}$$

En general:

$$\mathsf{N}(\delta) = \begin{cases} 0 & \text{si } \varepsilon < 0 & \text{acortamiento} \\ \mathsf{K}\delta & \text{si } 0 \leq \varepsilon \leq \varepsilon_e & \text{e. proporcional} \\ \sigma_{\mathbf{u}}\mathsf{A} & \text{si } \varepsilon_e \leq \varepsilon \leq \varepsilon_{\mathbf{u}} & \text{e. plástico} \\ 0 & \text{si } \varepsilon_{\mathbf{u}} < \varepsilon & \text{rotura} \end{cases}$$

Sólido deformable: cables

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GNULinux/LaTeX/dvips/ps2pdf

Si no se lo creen...